NRL Memorandum Report 2418

Non-Physical Self Forces in Electromagnetic Plasma-Simulation Algorithms

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March 1972

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Washington, D.C.

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Research Laboratory Washington, D.C. 20390	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP ---

3. REPORT TITLE

NON-PHYSICAL SELF FORCES IN ELECTROMAGNETIC PLASMA SIMULATION ALGORITHMS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (First name, middle initial, last name)

Jay P. Boris and Roswell E. Lee

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 1972	14	10

8a. CONTRACT OR GRANT NO. NRL Problem No. 77H02-27 b. PROJECT NO. DNA Subtask No. HC 04001 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Memorandum Report 2418 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release; distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Defense Nuclear Agency Washington, D.C. 20305

13. ABSTRACT

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Plasma simulation algorithms						
Electromagnetic plasma simulation						
Self forces						

ABSTRACT

A simple algorithm is described here for removing nonphysical self forces from two popular electromagnetic plasma simulation models. This algorithm also has two additional features; it considerably reduces short-wavelength noise and unwanted numerical fluctuations, and permits faster integration of the particle orbit equations by roughly a factor of two.

PROBLEM STATUS

This is an interim report on a continuing problem.

AUTHORIZATION

NRL Problem H02-27
DNA Subtask HC 04001

~~This note~~ describes a simple algorithm for removing nonphysical self forces from two popular electromagnetic plasma simulation models. This algorithm also has two additional features; it considerably reduces short-wavelength noise and unwanted numerical fluctuations, and permits faster integration of the particle orbit equations by roughly a factor of two. It is currently being included in the CLYRAD code [1,2].

There are three major numerical models in which electromagnetic radiation fields are self-consistently coupled to the Lorentz-Newton equations for the charged-particle motion. The first of these, proposed by Buneman [3], **includes an explicit leapfrog algorithm for solving the** Maxwell Equations and a special charge-current algorithm, based on the NGP (Nearest Grid Point) particle interpolation, which ensures that the continuity equation for charge and current is automatically satisfied at each timestep in finite-difference form. The second of these three algorithms [1] uses essentially the same field treatment but has a more flexible treatment of current accumulation in which, however, a Poisson Equation must be solved. The third algorithm [1] differs from the first two in the treatment of the electromagnetic fields. Rather than a finite difference approximation to the Maxwell Equations, the algorithm solves the Fourier transform of the equations in $\underset{\sim}{k}$-space. This third algorithm has not been implemented in multidimensions but a fore-runner method in one dimension was devised by Langdon and Dawson [4].

Many variations of the first two algorithms are possible and several have been coded and used [5,6]. Two of these are the Morse-Nielson algorithms A and B. Their algorithm A is an important generalization of Buneman's algorithm to the PIC (particle-in-cell) linear interpolation [8,9]. This algorithm also satisfies Poisson's

Equation automatically at every timestep. The Morse-Nielson algorithm B, as pointed out by Langdon [7], is essentially equivalent to the Boris algorithm. Thus our comments apply to the Boris algorithm used in CYLRAD, to both Morse-Nielson algorithms and to the Sinz [5] algorithm.

These electromagnetic algorithms treat particles moving on staggered, interlaced grids of variables as shown in Fig. 1 for two dimensions. Each field variable is so positioned that the time-dependent Maxwell Equations reduce to an extremely simple, finite-difference form for advancing $\underset{\sim}{E}$ and $\underset{\sim}{B}$ which is reversible and second-order accurate. The generalization of this 2D field-variable arrangement to 3D is straight forward and the specialization to 1D is trivial. As can be seen, the finite-difference form of the divergence equation which should be satisfied at each cycle is

$$\frac{(E_x(i+1/2,j) - E_x(i-1/2,j))}{\delta x} + \frac{(E_y(i,j+1/2) - E_y(i,j-1/2))}{\delta y}$$

$$= 4\pi\rho(i,j). \qquad (1)$$

The electrostatic fields which result from placing a particle at rest on the (i,j) grid point are different from those found in an electrostatic code because the x and y electric field grids are displaced, as seen in Fig. 1, by $\delta x/2$ and $\delta y/2$ respectively from the positions they would occupy in a standard electrostatic code [8,9]. To simplify the analysis, the electrostatic and the electromagnetic grids are compared in Fig. 2 for a 1D case where a test particle of unit charge is at

position α off a grid point ($0 < \alpha < \delta x/2 = 1/2$).

Consider first the electromagnetic grid as used in current simulation codes (Fig. 2). The linearly interpolated charge density at grid points 0 and 1 are $\rho(0) = 1-\alpha$ and $\rho(1) = \alpha$. If $\pm E_{1/2}$, $\pm E_{3/2}$ are defined to be the electric field at $x = \pm 1/2$ and $x = \pm 3/2$ respectively due to a unit charge at $x = 0$, then summing the contributions from the two grid points one gets

$$\begin{aligned} E_x(-1/2) &= -(1-\alpha)\, E_{1/2} - \alpha\, E_{1/2}, \\ E_x(1/2) &= (1-\alpha)\, E_{1/2} - \alpha\, E_{1/2}, \\ E_x(3/2) &= (1-\alpha)\, E_{3/2} + \alpha\, E_{1/2}. \end{aligned} \tag{2}$$

Since the particle lies between $E_x(1/2)$ and $E_x(-1/2)$, the linearly interpolated x electric field, as would be found by an electromagnetic simulation code, is

$$\begin{aligned} E_x(\alpha) &= (1/2+\alpha)\, E_x(1/2) + (1/2-\alpha)\, E_x(1/2) \\ &= 2\alpha\,(1-2\alpha) \left[E_{1/2} - \frac{E_{1/2}+E_{3/2}}{4}\right] \neq 0. \end{aligned} \tag{3}$$

Equation (3) shows that the self electrostatic field of a single simulation particle is non-zero. Thus all sorts of spurious effects can result. Figure 3 shows a 1D plot of the equivalent potential a particle would see due to its self-force. We have carried out tests on an electromagnetic code and the oscillations of a particle in this self field have been observed. The preceeding analysis has been generalized to electrostatic and magnetostatic self-forces in two and three dimensions. In every case the results are the same; spurious electrostatic and magnetostatic

forces are found when the charges do not exactly lie on grid lines. We know that there are no self forces in the usual electrostatic codes [8,9] and the electrostatic-code field in Fig. 2 are clearly closely related to the electromagnetic-code fields. Therefore, it should be a simple matter to eliminate the electrostatic self-forces from the electromagnetic-code. From Fig. 2 we have

$$E_x(1/2) = \frac{\Phi(1) - \Phi(0)}{\delta x} \tag{4}$$

and

$$E_x(0) = \frac{\Phi(1) - \Phi(-1)}{\delta x} \,. \tag{5}$$

the fields are therefore related by

$$E_x(0) = 1/2 \quad [E_x(1/2) + E_x(-1/2)] \tag{6}$$

If we linearly interpolate to the particle position using the averaged fields of Eq. (6) and Eq. (2), we get

$$\begin{aligned} E_x(\alpha) &= (1-\alpha)\, E_x(0) + \alpha\, E_y(1) \\ &= 0. \end{aligned} \tag{7}$$

This is the desired result of zero self-force. When generalized to two and three dimensions, the electrostatic and magnetostatic self forces are found to be zero. Furthermore, since the argument is basically one of symmetry rather than being based on any particular force law, the determination of $\Phi(i)$ from $\rho(i)$ admits all finite-sized particle algorithms

as well as the 3,5, and 7 point Poisson operators found in one, two, and three dimensions.

The averaging algorithm developed here for fully electromagnetic simulations in two dimensions is an obvious extension of Eq.(6) to the field-variable layout shown in Fig. 1. The three components of particle current density, $J_x^p(i,j)$, $J_y^p(i,j)$ and $J_z^p(i,j)$ are all found by linear interpolation onto the (i,j) grid, exactly as $\rho(i,j)$. The current densities for use on the interlaced field grids are then computed as follows:

$$
\begin{aligned}
J_x(i+1/2,j) &\equiv 1/2\ [J_x^p(i,j) + J_x^p(i+1,j)], \\
J_y(i,j+1/2) &\equiv 1/2\ [J_y^p(i,j) + J_y^p(i,j+1)], \qquad (6)\\
J_z(i,j) &\equiv J_z^p(i,j).
\end{aligned}
$$

The six field components, E_x, E_y, E_z, B_x, B_y, and B_z are then all integrated exactly as prescribed by the staggered-leapfrog algorithm. These field components are, however, averaged back to the particle grid before being used to advance the particle equations of motion. Thus

$$
\begin{aligned}
E_x^p(i,j) &\equiv 1/2\ [E_x(i+1/2i,j) + E_x(i-1/2,j)] \\
E_y^p(i,j &\equiv 1/2\ [E_y(i,j+1/2) + E_y(i,j-1/2)] \\
E_z^p(i,j) &\equiv E_z(i,j), \\
B_x^p(i,j) &\equiv 1/2\ [B_x(i,j+1/2) + B_x(i,j-1/2)], \qquad (9)\\
B_y^p(i,j) &\equiv 1/2\ [B_z(i+1/2,j) + B_z(i-1/2,j)], \\
B_z^p(i,j) &\equiv 1/4\ [B_z(i+1/2,i+1/2) + B_z(i+1/2,j-1/2) \\
&\qquad + B_z(i-1/2,j+1/2) + B_z(i-1/2,j-1/2)].
\end{aligned}
$$

The original fields $\underset{\sim}{E}$ and $\underset{\sim}{B}$ are retained unchanged, however, for use in the next cycle to advance Maxwell's Equations. They are not to be computed as averages of the particle fields $\underset{\sim}{E}^p$ and $\underset{\sim}{B}^p$.

The immediate consequence of adding the averaging stages given by Eqs. (8) and (9) to standard fully electromagnetic codes is the removal of nonphysical electrostatic and magnetostatic self-forces. There are two other favorable and important consequences. First, the averages required are smoothing operations; therefore, spurious numerical Cherenkov radiation and bremmstrahlung, arising mostly at short wavelengths should be strongly suppressed. Second, a sizeable simplification results since all particle quantities are now defined on a single grid. Only one set of bilinear weight coefficients need be found, rather than four, and thus it is expected that optimized particle integration can be speeded up by at least a factor of two.

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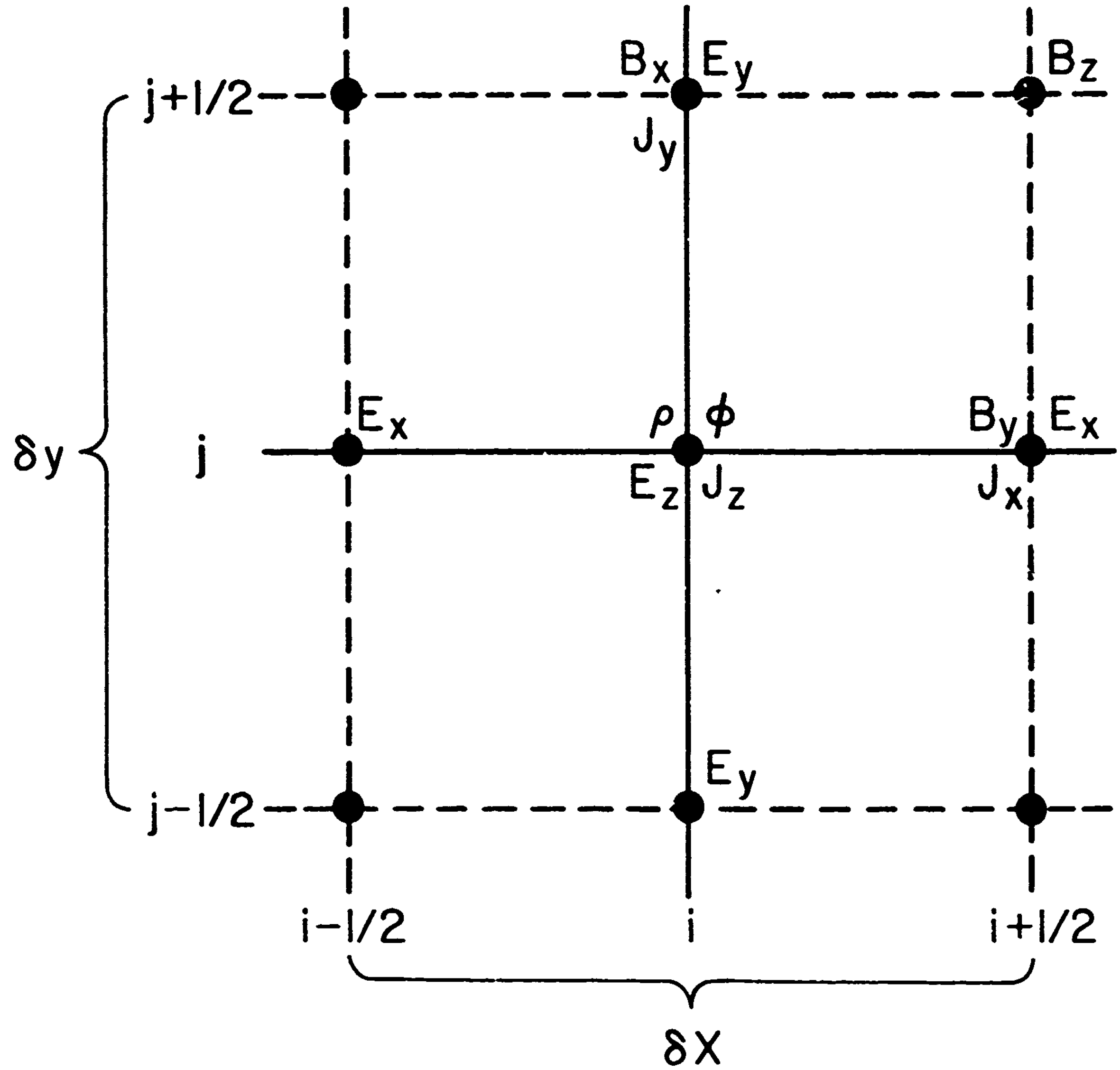

Fig. 1 - The staggered-interlaced grids of a 2D electromagnetic simulation code. In three dimensions the E_z, J_z, B_x, and B_y grids are all displaced half a cell, $\delta z/2$ out of the plane of the figure.

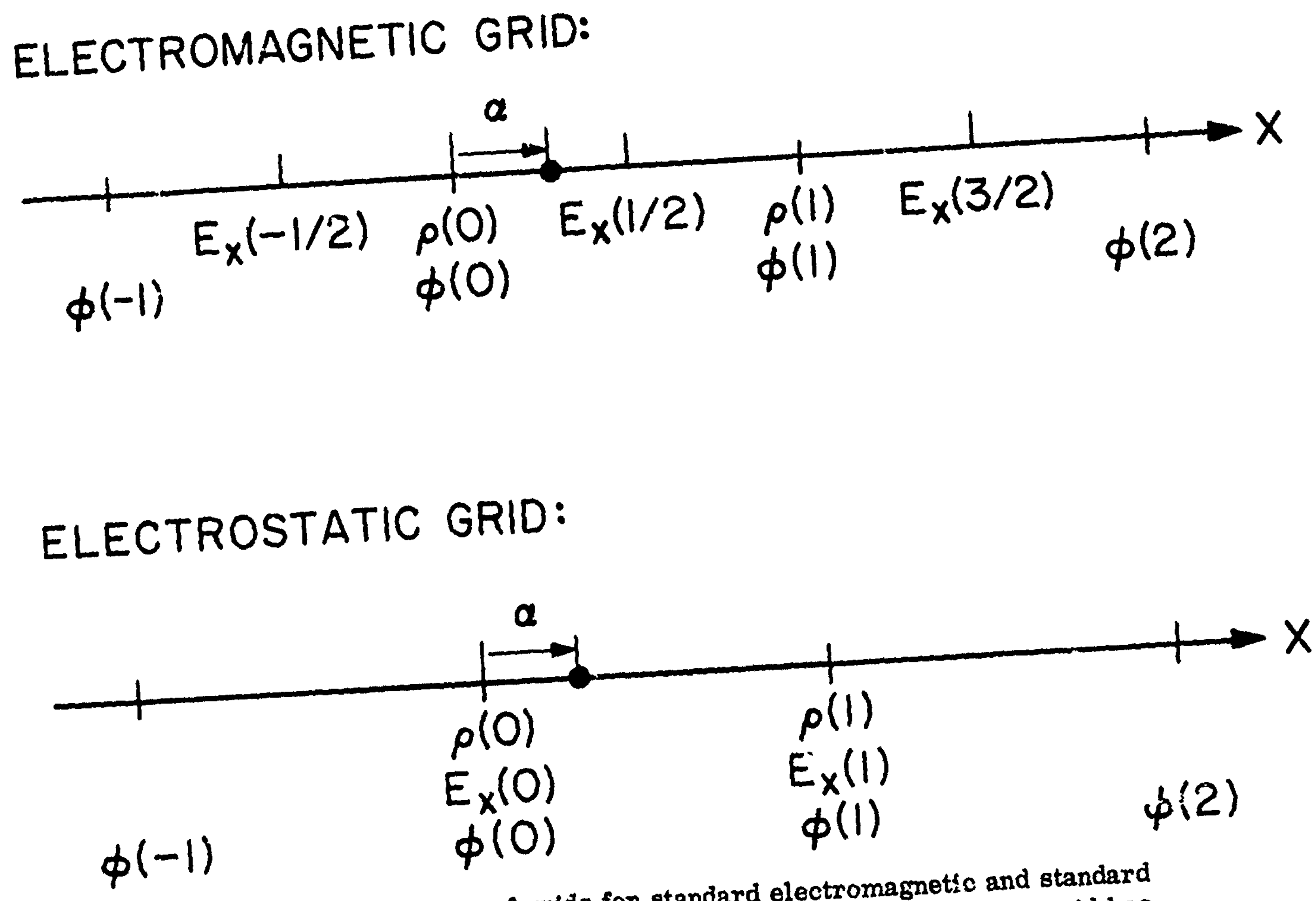

Fig. 2 - Comparison of grids for standard electromagnetic and standard electrostatic algorithms in 1D. The staggered electromagnetic grid has electric fields defined at points half a cell removed from the points of charge-density definition.

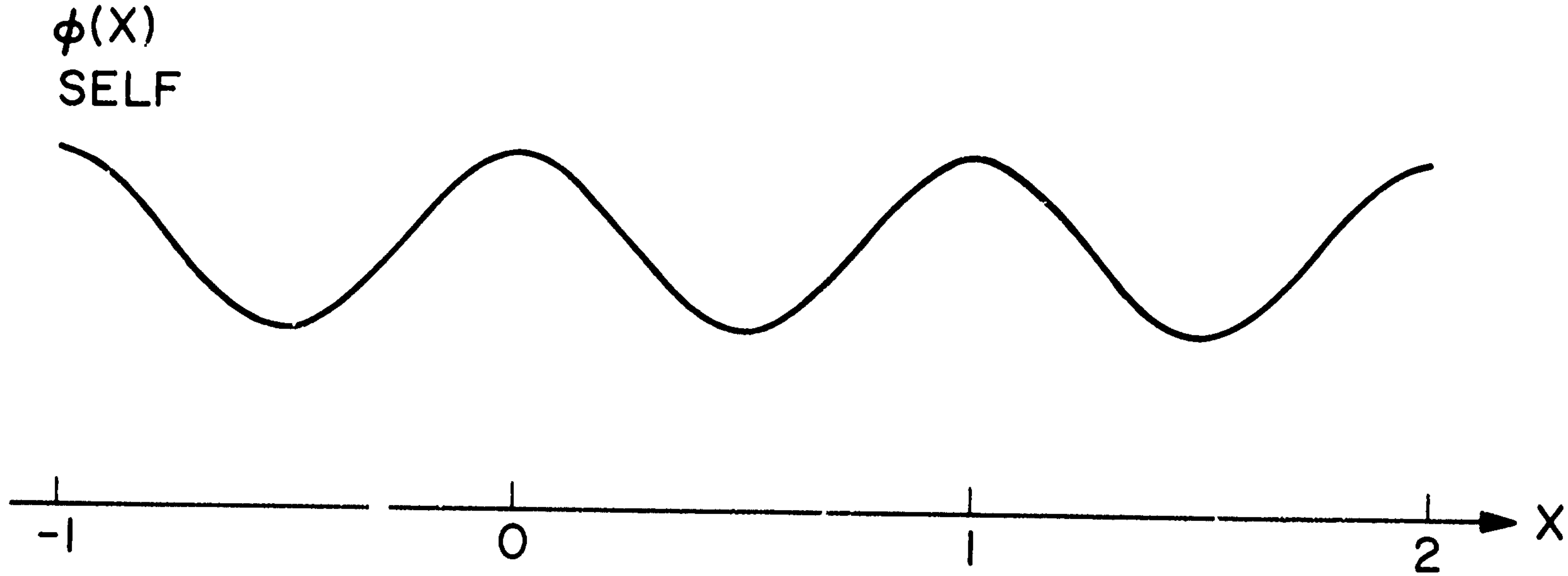

Fig. 3 – The electrostatic self pctential of a particle on a staggered 1D electromarnetic grid.